ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 10 NOTES

Unit 10: Practical Geometry

Short notes on bisectors, medians and altitudes, the four centres of a triangle, the 2 : 1 centroid rule and when a triangle can be constructed.

Bisectors | Four centres | Centroid | Constructing triangles | Ambiguous case

1. Bisectors
2. Four centres
3. Where the centres lie
4. Medians and the centroid
5. Equilateral triangle
6. Constructing triangles
7. The ambiguous case
8. Common mistakes
9. Quick check

1 · LINES IN A TRIANGLE

Bisectors, medians and altitudes

Angle bisector. A ray that divides an angle into two equal parts.

Right (perpendicular) bisector. A line through the **mid-point** of a segment that is also **perpendicular** to it. A segment has many bisectors, but only one is perpendicular.

Median. Joins a vertex to the mid-point of the opposite side. It always lies inside the triangle.

Altitude. A perpendicular from a vertex to the opposite side (extended if necessary).

Do not mix them up. Angle bisectors and right bisectors of sides are different lines with different jobs. A line through a mid-point is a right bisector only if it is also perpendicular.

2 · POINTS OF CONCURRENCY

Four special points

Lines that pass through one common point are **concurrent**. Points on one line are **collinear**.

In-centre. Where the three **angle bisectors** meet. It is always inside the triangle.

Circumcentre. Where the **right bisectors of the sides** meet. It is equally far from all three vertices, so it is the centre of the circle through them.

Centroid. Where the **medians** meet. Always inside. It is the centre of gravity of a triangular body.

Orthocentre. Where the **altitudes** meet.

3 · POSITION OF THE CENTRES

Inside, on or outside?

Acute-angled triangle. Altitudes meet inside, and the circumcentre is inside.

Right-angled triangle. The two arms are themselves altitudes, so the orthocentre is the **right-angle vertex**. The circumcentre is the **mid-point of the hypotenuse**.

Obtuse-angled triangle. The altitudes (extended) and the right bisectors of the sides meet **outside** the triangle.

Always true. The in-centre and centroid are inside every triangle. The circumcentre, orthocentre and centroid lie on one straight line.

4 · THE CENTROID

The 2 : 1 rule

$$\text{vertex to centroid} : \text{centroid to mid-point} = 2 : 1$$

The part from the vertex is twice the part from the mid-point. So the centroid is $2/3$ of the way along the median from the vertex.

A median is 18 cm long. The centroid is $18 \times 2/3 = \mathbf{12\ cm}$ from the vertex. For a 15 cm median, $15 \times 2/3 = \mathbf{10\ cm}$.

If the longer part of a median is 8 cm, the shorter part is $8 \div 2 = \mathbf{4\ cm}$.

5 · A SPECIAL TRIANGLE

Everything coincides

In an **equilateral triangle**, the in-centre, circumcentre, orthocentre and centroid are all the **same point**.

From each vertex, the altitude, median, angle bisector and right bisector of the opposite side are all the same line.

So you can find the in-centre without bisecting angles: the centroid, found with medians, is the same point.

The four centres coincide **only** in an equilateral triangle.

6 · CONSTRUCTING TRIANGLES

How much information is enough?

Three sides (that can form a triangle): exactly **one** triangle.

Two sides and the included angle: one triangle.

Two angles and any side: one triangle. Find the third angle from the angle sum 180°, then construct. For example, 45° and 60° give a third angle of 75°. Angles 60° and 90° give 30°.

Two sides and a non-included angle is the **ambiguous case**. There may be two triangles, one triangle, or none.

7 · TWO SIDES AND A NON-INCLUDED ANGLE

Compare with the shortest distance

Given sides *a* and *b* with the angle opposite side *a*, work out $h = b\sin(\text{angle})$, the shortest distance from the end of side *b* to the other arm. Then compare *a* with *h* and *b*.

***a* < *h*: no triangle.** Sides 2.3 cm and 5.4 cm, angle 30° opposite 2.3 cm. $h = 5.4 \times \sin 30° = 2.7$, and 2.3 < 2.7. The arc cannot reach the arm. Likewise 3 cm and 8 cm with 30°: *h* = 4, and 3 < 4.

***h* < *a* < *b*: two triangles.** Sides 6.1 cm and 3.8 cm, angle 30° opposite 3.8 cm. $h = 6.1 \times \sin 30° = 3.05$, and 3.05 < 3.8 < 6.1, so the arc cuts the arm twice. Sides 4.8 cm and 4.3 cm with 60°: $h \approx 4.16 < 4.3 < 4.8$, so two triangles.

Obtuse angle opposite the longer side: the arc cuts the arm once, so **one** triangle. Sides 5.7 cm and 7.5 cm with 105° opposite 7.5 cm gives one triangle.

Right angle. If the given angle is 90°, the side opposite it is the hypotenuse and must be the longest side. Then exactly one triangle exists.

8 · COMMON MISTAKES

Five traps to avoid

Mixing up the centres. Angle bisectors give the in-centre, right bisectors the circumcentre, medians the centroid, altitudes the orthocentre.

Expecting every centre inside the triangle. In an obtuse triangle, the circumcentre and orthocentre are outside.

Reversing the 2 : 1 ratio. The longer part is next to the vertex.

Treating any bisector as a right bisector. It must also be perpendicular.

Assuming two sides and any angle give one triangle. Only the included angle does.

9 · QUICK CHECK

Try these, then check your answers

Where do the altitudes of a right-angled triangle meet?

Answer: At the vertex of the right angle.

A median is 18 cm long. How far is the centroid from the vertex?

Answer: 18 × 2/3 = 12 cm.

Two angles of a triangle are 45° and 60°. What is the third?

Answer: 180° − 45° − 60° = 75°.

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