ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 2 NOTES

Unit 2: Logarithms

Short notes on scientific notation, logarithms, their laws, change of base and common mistakes.

Scientific notation | Definition | Characteristic and mantissa | Laws | Change of base

1. Scientific notation
2. What a logarithm is
3. When it is defined
4. Common and natural logs
5. Characteristic and mantissa
6. Laws of logarithms
7. Change of base
8. Using logarithms
9. Common mistakes
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1 · SCIENTIFIC NOTATION

Write big and small numbers neatly

$$d \times 10^{n}$$

$1 \le d < 10$ | n is an integer

Big number

$4500 = 4.5 \times 10^{3}$ point moves 3 places left

Small number

$0.00045 = 4.5 \times 10^{-4}$ point moves 4 places right, so the exponent is negative

Not scientific notation

12.5×10^{3} has $d = 12.5$, which is too big. Write 1.25×10^{4}.

2 · LOGARITHMS

A logarithm is an exponent

$$\log_b x = y \iff b^y = x$$

Read it as: **the power to which *b* must be raised to get *x* is *y*.**

$\log_2 8 = 3$ because $2^3 = 8$

$\log_{10} 1000 = 3$ because $10^3 = 1000$

$\log_9 3 = ½$ because $9^{1/2} = 3$

3 · RULES FOR A VALID LOGARITHM

$\log_b x$ needs three conditions

$b > 0$ $b \neq 1$ $x > 0$

Why not a negative number?

$\log_5(-1)$ would need $5^y = -1$, but a positive base never gives a negative result.

Why not base 1?

$\log_1 2$ would need $1^y = 2$, but 1 raised to any power is 1.

Two values to remember

$\log_b 1 = 0$ since $b^0 = 1$

$\log_b b = 1$ since $b^1 = b$

Example: $\log_2(4 - 2x)$ is undefined at x = 2, because the bracket becomes 0.

4 · TWO SPECIAL BASES

Common, natural and antilog

Common logarithm

Base 10

Written $\log x$. Also called Briggsian logarithms.

Natural logarithm

Base *e*

Written $\ln x$, where $e \approx 2.71828$ (irrational). $\ln e = 1$ and $\ln 1 = 0$.

Antilogarithm

The antilog undoes the log: if $\log x = y$, then $x = \text{antilog}\, y$.

5 · PARTS OF A COMMON LOGARITHM

Characteristic and mantissa

$$\log x = \text{characteristic} + \text{mantissa}$$

Characteristic is the integral part. If $x = d \times 10^n$, it equals n. For $0.04 = 4 \times 10^{-2}$ it is -2.

Mantissa is the fractional part. It is never negative and is always less than 1.

Bar notation. $\log 0.0123 = \bar{2}.0899$ means $-2 + 0.0899$. Only the 2 is negative; the mantissa stays positive.

Counting digits. A whole number has $\text{characteristic} + 1$ digits. For example, 1009 has characteristic 3 and 4 digits.

6 · LAWS OF LOGARITHMS

Three laws turn hard sums into easy ones

Product law

$\log_b(mn) = \log_b m + \log_b n$

Quotient law

$\log_b(m/n) = \log_b m - \log_b n$

Power law

$\log_b(m^n) = n \log_b m$

Example. Given log 2 = 0.3010 and log 3 = 0.4771, find log 12.

$12 = 2^2 \times 3$, so $\log 12 = 2(0.3010) + 0.4771 =$ **1.0791**

Single logarithm. $5 \log 2 - 2 \log 5 = \log 32 - \log 25 = \log(32/25)$

7 · CHANGE OF BASE

Move a logarithm to a new base

$$\log_b m = \frac{\log_a m}{\log_a b}$$

Use a calculator. $\log_7 20 = \log 20 \div \log 7$, using base 10 for both.

Reciprocal rule. $\log_a b = 1 \div \log_b a$, so the two multiply to 1.

Example. $\log_2 3 \times \log_3 4 = (\log 3 \div \log 2) \times (\log 4 \div \log 3) = \log_2 4 =$ **2**

8 · USING LOGARITHMS

Where logarithms help

How many digits in 2^{20}?

$\log 2^{20} = 20 \times 0.3010 = 6.020$. The characteristic is 6, so the number has **7 digits**.

Richter scale. $M = \log(I \div I_0)$. If the intensity equals I_0, then M = log 1 = 0.

Comparing earthquakes. Each extra unit multiplies intensity by 10. Magnitude 7 against 5 is a difference of 2, so $10^2 =$ **100** times stronger.

9 · COMMON MISTAKES

Five traps to avoid

Splitting a sum. $\log(50 - 5) \neq \log 50 - \log 5$. The laws handle products and quotients only, never sums or differences inside the bracket.

Confusing a quotient. $\log_{10} x \div \log_{10} y$ is $\log_y x$, not $\log(x/y)$.

Skipping the check. After solving $\log(x + 3) = \log(15x - 4)$ you get x = 0.5. Both brackets must be positive: each equals 3.5, so it is valid.

Wrong sign for a base. If $\log_{\sqrt{x}} 25 = 4$, then x = 5 or −5, but a base must be positive, so x = 5.

Positive exponent for small numbers. 0.04 = 4×10^{-2}, so its characteristic is −2, not 2.

10 · QUICK CHECK

Try these, then check your answers

Find x if $\log_3 x = 2$.

Answer: $x = 3^2 = 9$.

Evaluate $\log_9 3$.

Answer: ½, because $9^{1/2} = 3$.

Write $5 \log 2 - 2 \log 5$ as one logarithm.

Answer: log(32/25).

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