ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 5 NOTES

Unit 5: Linear Equations and Inequalities

Short notes on linear equations, absolute value, radical equations, linear inequalities and compound inequalities.

Linear equations | Absolute value | Radical equations | Inequalities | Compound inequalities

1. Linear equations
2. Solving equations
3. No solution
4. Absolute value
5. Absolute value equations
6. Radical equations
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1 · LINEAR EQUATIONS

One variable, power one

$$ax + b = 0,\ a \neq 0$$

Degree one. The exponent of the variable is always 1. So $x^2 = 4$ is not linear, and neither is $xy = 14$ (a product of two variables).

Solution. A value of the variable that makes both sides equal.

Equivalent equations have the same solutions. Adding or subtracting the same number on both sides keeps them equivalent.
$2x + 3 = 4 \Leftrightarrow 2x = 1$

2 · SOLVING

Clear fractions and decimals first

$-7x + 24 = 3$
Subtract 24: $-7x = -21$. Divide by -7: **$x = 3$**.

Fractions. $3x/5 - 1/2 = x/4 + 1$
Multiply every term by 20, the LCM of the denominators, so every term becomes whole: $12x - 10 = 5x + 20$. Then $7x = 30$ and **$x = 30/7$**.

Decimals. $0.7(x - 1) - 0.5x = 1.1$
Expand: $0.2x - 0.7 = 1.1$, so $0.2x = 1.8$ and **$x = 9$**.

3 · SPECIAL CASES

When there is no solution

$2x + 3 = 2x + 5$

The variable cancels and leaves $3 = 5$, which is false. No value of x works.

We then write the solution set as φ, the empty set.

4 · ABSOLUTE VALUE

Distance from zero

$|x|$ is the distance of x from 0, so it is never negative. $|-5| = 5$.

Two cases. $|x| = x$ when $x \geq 0$, and $|x| = -x$ when $x < 0$.

Products. $|ab| = |a|\,|b|$

Equal absolute values. If $|a| = |b|$, then $a = b$ or $a = -b$, since both are the same distance from 0.

$|x| = 0$ has exactly one solution, $x = 0$.

5 · ABSOLUTE VALUE EQUATIONS

Solve two equations

$$|ax + b| = c \quad (c > 0)$$

The quantity inside the bars can be c or $-c$, so solve $ax + b = c$ and $ax + b = -c$.

$|2x + 5| = 11$

$2x + 5 = 11$ gives $x = 3$. $2x + 5 = -11$ gives $x = -8$.

$|3x + 2| = 7$

$3x + 2 = 7$ gives $x = 5/3$. $3x + 2 = -7$ gives $x = -3$.

Negative on the right. $|x| = -6$ has no solution. Likewise $|x| + 7 = 3$ gives $|x| = -4$, which is impossible.

6 · RADICAL EQUATIONS

Square, then always check

A **radical equation** has the variable under a radical sign, such as $\sqrt{(x+1)} = 7$.

$\sqrt{(2x+1)} = 9$
Square both sides: $2x + 1 = 81$, so x = 40. Check: √81 = 9 ✓.

$\sqrt{x} + 3 = 7$
Isolate the root: $\sqrt{x} = 4$, so x = 16. Check: √16 + 3 = 7 ✓.

Extraneous roots. Squaring can create a root that fails the original equation. Such a root is called **extraneous** and is rejected.

No solution. $\sqrt{(3y+1)} = -2$ is impossible, because a principal square root is never negative. In $4 + 2\sqrt{(3y+1)} = 3$, squaring gives y = −¼, but that fails the check, so there is no solution.

7 · INEQUALITIES

Solving linear inequalities

A linear inequality in one variable has the form $ax + b < 0$ (or >, ≤, ≥) with $a \neq 0$. **Strict** inequalities use only < and >.

$x + 5 < 8$: subtract 5 from both sides, giving $x < 3$.

$3x - 2 < 7$ gives $3x < 9$, so $x < 3$. The value 3 is not a solution, because at x = 3 the left side equals 7, which is not less than 7.

8 · THE KEY RULE

Negative number? Flip the sign

When you multiply or divide both sides of an inequality by a **negative** number, the inequality sign reverses.

$-2x < 6$
Divide by −2 and flip: **$x > -3$**.

$-3x \geq 12$
Divide by −3 and flip: **$x \leq -4$**.

The usual slip. Writing $x < -3$ for the first one. Check with a number: x = 0 satisfies −2(0) < 6, but 0 < −3 is false.

9 · COMPOUND INEQUALITIES

“And” and “or”

and

A number must satisfy **both**. The solution is the common part.

or

One is **enough**. The solution joins the two sets.

And. $x - 5 \geq -1$ and $x + 3 \leq 10$ give $x \geq 4$ and $x \leq 7$, so **$4 \leq x \leq 7$**.

Or. $2x + 3 > 7$ or $4x - 1 < 3$ give $x > 2$ or $x < 1$. Join the two sets.

Combined form. $4 < 2x + 3 < 10$ means two inequalities joined by “and”. For “twice a number plus 3 is between 5 and 17”: $5 < 2x + 3 < 17$, so **$1 < x < 7$**.

10 · COMMON MISTAKES

Five traps to avoid

Not flipping the sign after dividing by a negative number.

Giving one answer for $|x| = k$ when $k > 0$. There are two: k and $-k$. And if $k < 0$ there are none.

Skipping the check in a radical equation. Squaring can introduce an extraneous root.

Including the end value of a strict inequality. For $3x - 2 < 7$, $x = 3$ is not a solution.

Mixing up “and” with “or”. “And” keeps only the overlap; “or” keeps everything in either set.

11 · QUICK CHECK

Try these, then check your answers

Solve $|2x + 5| = 11$.

Answer: $x = 3$ or $x = -8$.

Solve $\sqrt{(2x + 1)} = 9$.

Answer: $x = 40$, since $2x + 1 = 81$. It passes the check.

Solve $-2x < 6$.

Answer: $x > -3$. The sign flips when dividing by -2.

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