ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 6 NOTES

Unit 6: Trigonometry and Bearing

Short notes on angle measure, arc length and sector area, the unit circle, trigonometric ratios, identities, elevation and depression, and bearings.

Radians | Arc and sector | Unit circle | Identities | Elevation | Bearings

1. Measuring angles
2. Arc and sector
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4. The unit circle
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1 · ANGLE MEASURE

Degrees and radians

Sexagesimal system. 1° = 60′ and 1′ = 60″, so 1° = 3600″. For example $60°30' = 60 + 30/60 = 60.5°$.

Direction. An anticlockwise turn from the initial side gives a **positive** angle. A clockwise turn gives a **negative** angle.

One radian is the central angle of an arc whose length equals the radius. A full turn is $360° = 2\pi$ radians.

Convert. π rad $= 180°$, so 1 rad = (180/π)° ≈ 57.2958°.
$45° = 45 \times \pi/180 = \pi/4$ · $2\pi/5 = 2\pi/5 \times 180/\pi = 72°$

2 · ARC LENGTH AND SECTOR AREA

Radians make the formulas simple

Arc length

$$l = r\theta$$

Sector area

$$A = \tfrac{1}{2}r^2\theta$$

θ must be in **radians**, because a radian is the ratio of arc to radius. Put $\theta = 2\pi$ and you get $l = 2\pi r$, the circumference.

Arc. r = 6 cm, angle 120° = 2π/3 rad. $l = 6 \times 2\pi/3 =$ **4π cm**

Sector. r = 12 cm, θ = π/3. $A = \tfrac{1}{2} \times 144 \times \pi/3 =$ **24π cm^2**

For a fixed radius, the sector area is directly proportional to the angle. Double the angle and the area doubles.

3 · ANGLES IN STANDARD POSITION

Quadrantal and co-terminal angles

Quadrantal angle. The terminal ray lies on an axis: 0°, 90°, 180°, 270°, 360° and so on, all multiples of 90°. So 45° is not quadrantal.

Co-terminal angles share the same terminal ray, so they differ by a whole number of full turns (360°). $390° - 30° = 360°$ and $60° - (-300°) = 360°$.

Co-terminal angles have the **same trigonometric ratios**, because the ratios depend only on where the terminal ray meets the unit circle.

4 · THE UNIT CIRCLE

Where sine and cosine come from

$$x^2 + y^2 = 1$$

The point at angle θ is $\mathrm{P}(\cos\theta, \sin\theta)$. So $\cos\theta = x$ and $\sin\theta = y$.

A point on the unit circle is never more than 1 unit from the axes, so $\sin\theta$ is never greater than 1.

$\tan\theta = y/x$. At 90° the point is (0, 1), so $\tan 90° = 1/0$ is **undefined**.

$\operatorname{cosec}\theta = 1/\sin\theta$, so $\operatorname{cosec} 30° = 1/(1/2) = 2$ and $\operatorname{cosec} 0°$ is undefined because sin 0° = 0.

5 · VALUES TO KNOW

Special angles

sin

$\sin 0° = 0$ | $\sin 30° = 1/2$ | $\sin 45° = 1/\sqrt{2}$ | $\sin 60° = \sqrt{3}/2$ | $\sin 90° = 1$

cos

$\cos 0° = 1$ | $\cos 30° = \sqrt{3}/2$ | $\cos 45° = 1/\sqrt{2}$ | $\cos 60° = 1/2$ | $\cos 90° = 0$

tan = sin ÷ cos

$\tan 30° = 1/\sqrt{3}$ | $\tan 45° = 1$ | $\tan 60° = \sqrt{3}$

On the axes. $\cos 180° = -1$ (point (−1, 0)) and $\sin 270° = -1$ (point (0, −1)).

Example. $\sin 60° \cos 30° - \cos 60° \sin 30° = 3/4 - 1/4 =$ **1/2**

6 · SIGNS

Which ratio is positive where?

Quadrant I

All ratios positive.

Quadrant II

Only sin (and cosec) positive. $y > 0$ but $x < 0$.

Quadrant III

Only tan (and cot) positive. Both x and y are negative, so sin and cos are negative but y/x is positive.

Quadrant IV

Only cos (and sec) positive.

Example. If $\cos\theta = -3/5$ and θ is in quadrant II, then sin θ = 4/5 (positive). So $\tan\theta = (4/5) \div (-3/5) =$ **$-4/3$**.

7 · IDENTITIES

True for every angle

$\sin^2\theta + \cos^2\theta = 1$

It comes from $x^2 + y^2 = 1$, so it holds for **every** angle, not only acute ones.

$\tan^2\theta + 1 = \sec^2\theta$

Divide the first identity by $\cos^2\theta$.

Not an identity: $\sin\theta + \cos\theta = 1$. At 30° the sum is 1/2 + √3/2 ≈ 1.37. It happens to equal 1 at 0° and 90°, but that does not make it an identity.

8 · RIGHT-ANGLED TRIANGLES

Elevation and depression

$\tan\theta = \text{perpendicular} \div \text{base}$. If one acute angle is θ, the other is $90° - \theta$.

Angle of elevation: measured from the horizontal **up** to the line of sight. The **angle of depression** is measured **down** from the horizontal. Horizontal lines are parallel, so the depression of a boat from a tower equals the elevation of the tower from the boat.

Tree. 20 m from its foot, elevation 45°. $\text{height} = 20 \times \tan 45° =$ **20 m**

Tower. 60 m high, car seen at depression 60°. $\tan 60° = 60/d$, so $d = 60/\sqrt{3} =$ **$20\sqrt{3}$ m**.

Kite. String 50 m at 30° to the ground. $\text{height} = 50 \times \sin 30° =$ **25 m**

9 · BEARINGS

Direction as an angle from north

A **bearing** is the angle measured **clockwise from north**, written with three digits: north 000°, east 090°, south 180°, west 270°.

North-west lies halfway between west (270°) and north (360°), so its bearing is **315°**.

Back bearing. Seen from the other end, the line turns 180°. If the bearing of B from A is 040°, the bearing of A from B is $040° + 180° =$ **220°**.

Walking. 5 km north, then 5 km east: $\tan\theta = 5/5 = 1$, so the end point is at bearing **045°**.

A bearing of 120° lies between 090° (east) and 180° (south), so B is to the south-east of A.

10 · COMMON MISTAKES

Five traps to avoid

Degrees in the arc formula. $l = r\theta$ works only when θ is in radians.

Thinking the identity works only for acute angles. $\sin^2\theta + \cos^2\theta = 1$ holds for all angles.

Ignoring the quadrant. Check the sign of sin, cos and tan before you finish.

Writing tan 90° = ∞ as a number. It is undefined.

Bearings from the wrong direction. Measure clockwise from north, with three digits.

11 · QUICK CHECK

Try these, then check your answers

Convert 45° to radians.

Answer: 45 × π/180 = π/4.

The bearing of B from A is 040°. What is the bearing of A from B?

Answer: 040° + 180° = 220°.

In which quadrant are sin θ and cos θ both negative?

Answer: Quadrant III, where both x and y are negative.

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